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Constitutive models for anisotropic frictional heat

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Abstract—Frictional heat always accompanies the friction process. It can be assumed that anisotropic friction induces anisotropic frictional heat for crystals, composites and materials with microstructure. Using a thermodynamical approach, a general method for deriving constitutive equations for frictional heat is presented. The directionality of frictional heat is described by a heat intensity function depending on the sliding direction. Symmetries and particular anisotropic heat directions are investigated. Examples illustrate different types of frictional heat anistropies.

1. INTRODUCTION

Tests carried out and everyday practice show that almost all the energy dissipated by friction between two bodies sliding one on another turns into frictional heat and wear. The heat generated at or very close to the contact raises the temperature at the contact area, and it is transfered away into a bulk of rubbing bodies and into surroundings. There are two different temperatures relating to frictional heat: the average temperature of the sliding surfaces and the "flash" temperature, which is generated at individual surface asperities as they pass in and out of contact. Small areas of localized asperity contacts and a short work time make the heat supplied to the bodies from the microcontacts very small.

Temperatures of rubbing surfaces can be measured in a number of ways: with thermocouples, by infrared thermography, by use of temperature-sensitive coatings, etc. Measurements of the contact temperature have been carried out by many researches [1–5].

Theoretical investigations of contact thermomechanical problems were devoted mostly to contact temperature studies. There have been numerous attempts to estimate the temperature of sliding surfaces, e.g. Blok [6], Jaeger [7], Ling [8], Kennedy [9], Barber and Comninou [10] and others [11–13]. To this end, an adequate heat conduction problem was solved, using the methods heat source, integral transformation and finite elements. Several methods based on empirical relations and simplified theoretical assumptions were proposed for calculating the flash and average temperatures. In refs. [14–21] are given some of the most representative ones.

A heat flux at the contact zone is the sum of the frictional heat and the heat conducted from one body to another. Usually, constitutive relations for the conductive heat are based on observations that the heat flux across the contact depends on a temperature discontinuity. A constant of the constitutive relation, i.e. thermal contact conductance, is a function of pressure and microdeformations [10, 22]. Nonhomogeneous heat transfer across the contact and frictional heat can induce a change of the contact geometry (so-called thermoelastic instability of the contact) [10, 23, 24]. Another investigated problem is devoted to division (partitioning) of the frictional heat between the sliding members [25]. A directional dependence of the frictionally-generated surface temperature has been measured for composites. The referenced studies [1–25] cover all investigated problems within the frictional heat physics.

The field of temperature induced by friction is not an additional effect accompanying the friction process, but inseparable behaviour of this phenomenon. It can be presumed that anisotropic friction induces anisotropic heat generation, i.e. the intensity of the generated heat depends on the sliding direction. Anisotropy of friction results from roughness anisotropy of contacting surfaces and anisotropy of mechanical properties of composites, fibre-reinforced materials and crystals.

The contact of bodies represents an irreversible thermomechanical process. Unremovable changes (heating, abrasion, etc.) occur in the rubbing bodies, and supplied external work cannot be recovered by a simple change of the contact process direction to the opposite one. Therefore, each theoretical approach for the description of the contact phenomena has to be based on thermodynamical considerations. We believe that this subject may be studied and understood at different levels of generalization. The purpose of this

NOMENCLATURE

A, B	contacting bodies
c	specific heat
$\mathbf{c}$	couple of friction forces at the layer S
$\mathbf{C}_{ik}, C_k^{ij}$	friction tensors
F	contact area
g	pin circumference
G_w, G_w^A, G_w^{AB}	groups of symmetry
h	heat transfer coefficient
$\mathbf{J}_{\mathbf{u}_i}, \mathbf{J}_{\mathbf{s}_i}$	mirror reflection transformations
k	thermal conductivity
K	thermal diffusivity
$\mathbf{L}$	particle at the layer S
m_0	mass of the material point
n, m; n_i, m_i	constant parameters in constitutive equations
$\mathbf{n}^+, \mathbf{n}^-$	outward normal vectors with respect to S^+ and S^-
N_{AS}, N_{AB}	normal pressure
o	full orthogonal group
$\mathbf{q}_A^f, \mathbf{q}_B^f, q_A^f$; q_B^f, q_{AB}^f	frictional heat fluxes
q_f^*	frictional heat supplied to the layer S
$\mathbf{r}$	radius vector of the material point
R	space of real numbers
$\mathbf{R}$	orthogonal transformations
$\mathbf{R}_{\mathbf{n}}^{\gamma}$	rotation transformation
S^+, S^-	wearing surfaces
S	wear product interfacial layer
$\mathbf{t}_{AS}, \mathbf{t}_{BS}$; $\mathbf{t}_{AB}, t^i$	friction forces
t	time
T, T_0	temperatures
$\mathbf{T}_A$	Cauchy stress tensor
$\mathbf{v}_A, \mathbf{v}_B$	material velocities of the bodies
$\mathbf{v}_S$	translational velocity of the layer particles
$\mathbf{v}_{AS}, v_j$	unit vector of the sliding velocity
$\mathbf{v}_0, \mathbf{v}_0^A, \mathbf{v}_0^B$	unit vectors of reference directions at the contacting surfaces
$\mathbf{V}_{AS}, \mathbf{V}_{BS}$; V_{AS}, V_{BS}	sliding velocities
w_A, w_B; w_{AB}, w_k	frictional heat intensities
$\mathbf{x}_A, \mathbf{x}_B, \mathbf{x}_S$	position vectors of particles of the bodies and the layer
$\mathbf{X}_A, \mathbf{X}_B$	particles in the bodies A and B
z	distance from the contact surface.
Greek symbols	
$\alpha_v, \alpha_v^A, \alpha_v^B$	angles between the sliding direction and the reference directions
α_0	direction of the initial sliding velocity
β	deviation angle of the anistropic friction force from the sliding direction
$\beta_A, \beta_B, \beta^*$	energies spent on wear process of the bodies and the layer
$\theta_A, \theta_B, \theta_S$	absolute temperatures of the bodies and the layer
κ	composition coefficient
$\mu_\alpha, \mu_\alpha^\perp$; $\mu_{(k)}$	friction coefficients
ρ	mass density
φ	angle of the relative position of the contacting surfaces
χ	rotation tensor of the layer particles
ω	rotational velocity of the layer particles.
Other symbols	
$\mathbf{1}$	identity transformation
$-\mathbf{1}$	central inversion transformation.

paper is to make the first step towards the development of rational models of anisotropic frictional heat, and to investigate their properties. We develop equations which are both useful and physically sound.

2. GENERAL AND SIMPLIFIED CONSTITUTIVE EQUATIONS FOR FRICTION AND FRICTIONAL HEAT

In dry contact conditions solids rub and wear out (i.e. become abraded). The unintentional removal of solid material from rubbing surfaces, and the generation and circulation of free debris, are main features of the wear process. The loose particles form a thin wear-product layer at the contact area. We assume that every particle of the layer can translate and rotate about its own axis. Let a system of two bodies A and B and the two-dimensional, interfacial layer S be a model of rubbing and wearing solids, Fig. 1.

Analysing governing equations for the contacting bodies [26], the following dependent variables of the contact phenomena can be derived: friction force vectors between the bodies and the layer ($\mathbf{t}_{AS}, \mathbf{t}_{BS}$), and fluxes of the frictional heat supplied to the bodies,

$$\mathbf{q}_A^f \cdot \mathbf{n}^+ = q_A^f \quad \mathbf{q}_B^f \cdot \mathbf{n}^- = q_B^f. \tag{1}$$

Here, we neglect dependent variables related to the

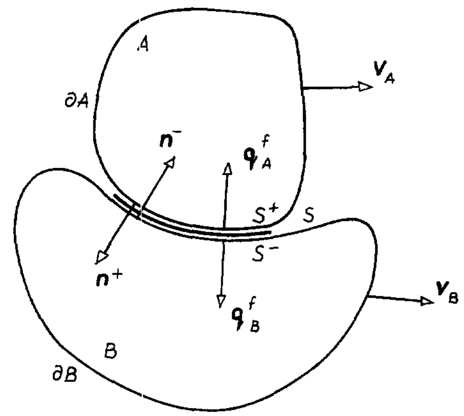


Fig. 1. Model of contacting and rubbing solids.

layer S (i.e. a couple of friction forces and frictional heat supplied to the layer) [26].

Let us assume that friction and frictional heat phenomena are generated at the contact, due to thermodynamical processes occuring in the contacting bodies and in the interfacial wear-product layer [26]. Thus, a relation exists between the dependent variables and motions of the bodies A and B ($\mathbf{x}_A(\mathbf{X}_A, t)$, $\mathbf{x}_B(\mathbf{X}_B, t)$), translational and rotational motions of the layer ($\mathbf{x}_S(\mathbf{L}, t), \chi(\mathbf{L}, t)$), and fields of temperature in the bodies and the layer ($\theta_A(\mathbf{X}_A, t)$, $\theta_B(\mathbf{X}_B, t)$, $\theta_S(\mathbf{L}, t)$). The functions are given for particles $\mathbf{X}_A, \mathbf{X}_B$ of the bodies A and B, particles $\mathbf{L}$ of the layer S and for time t. General constitutive equations for friction and frictional heat are formulated with the aid of the following functionals:

$$\{\mathbf{t}_{AS}, q_A^f\} = F_{AS}[\mathbf{x}_A(\mathbf{X}'_A, t'), \theta_A(\mathbf{X}'_A, t'), \mathbf{x}_S(\mathbf{L}', t'), \chi(\mathbf{L}', t'), \theta_S(\mathbf{L}', t'); \mathbf{X}_A, t] \quad (2)$$

$$\{\mathbf{t}_{BS}, q_B^f\} = F_{BS}[\mathbf{x}_B(\mathbf{X}'_B, t'), \theta_B(\mathbf{X}'_B, t'), \mathbf{x}_S(\mathbf{L}', t'), \chi(\mathbf{L}', t'), \theta_S(\mathbf{L}', t'); \mathbf{X}_B, t] \quad (3)$$

where

$$\mathbf{X}'_A \in S_0^+ \quad \mathbf{X}'_B \in S_0^- \quad \mathbf{L}' \in S_0 \quad t' \leqslant t \quad t' \in R. \quad (4)$$

Subscript zero means a reference configuration of the bodies and the layer.

The axioms of neighbourhood, memory and objectivity restrict the class of functions which may constitute functionals and their independent variables [26]. With respect to the neighbourhood and memory axioms, the functionals may be approximated by the functionals of gradients with respect to spatial and time variables,

$$\begin{aligned}\{\mathbf{t}_{AS}, q_A^f\} = F_{AS}[&\mathbf{x}_A(\mathbf{X}_A, t), \operatorname{Grad}\mathbf{x}_A(\mathbf{X}_A, t),\\ &\dot{\mathbf{x}}_A(\mathbf{X}_A, t), \theta_A(\mathbf{X}_A, t), \operatorname{Grad}\theta_A(\mathbf{X}_A, t),\\ &\dot{\theta}_A(\mathbf{X}_A, t), \mathbf{x}_S(\mathbf{L}, t), \operatorname{Grad}_S\mathbf{x}_S(\mathbf{L}, t),\\ &\dot{\mathbf{x}}_S(\mathbf{L}, t), \chi(\mathbf{L}, t), \operatorname{Grad}_S\chi(\mathbf{L}, t), \dot{\chi}(\mathbf{L}, t),\\ &\theta_S(\mathbf{L}, t), \operatorname{Grad}_S\theta_S(\mathbf{L}, t), \dot{\theta}_S(\mathbf{L}, t); \mathbf{X}_A, t].\end{aligned} \quad (5)$$

The axiom of objectivity reduces the class of independent variables and forms of the constitutive functionals that may be used for expressing the constitutive equation. A relative velocity of contacting particles ($\mathbf{V}_{AS}$), its unit vector ($\mathbf{v}_{AS}$) and norm (V_{AS}) satisfy the axiom of objectivity,

$$\mathbf{V}_{AS} = \dot{\mathbf{x}}_A(\mathbf{X}_A, t) - \dot{\mathbf{x}}_S(\mathbf{L}, t) \quad (6)$$

$$\mathbf{v}_{AS} = \frac{\mathbf{V}_{AS}}{V_{AS}} \quad V_{AS} = |\mathbf{V}_{AS}|. \quad (7)$$

A normal pressure between the body A and the layer S is the objective scalar

$$N_{AS} = |(\mathbf{n}^+ \otimes \mathbf{n}^+)(\mathbf{T}_A \mathbf{n}^+)|. \quad (8)$$

$\mathbf{T}_A$ is the Cauchy stress tensor in the body A,

$$\mathbf{T}_A = \hat{\mathbf{T}}_A(\operatorname{Grad}\mathbf{x}_A, \theta_A; \mathbf{X}_A). \quad (9)$$

Using equations (6)–(8), we may introduce simplified forms of the constitutive equations,

$$\begin{aligned}\mathbf{t}_{AS}(\mathbf{X}_A, t) &= \hat{\mathbf{t}}_{AS}(\mathbf{v}_{AS}, N_{AS}, \theta_A, \theta_S, \operatorname{Grad}\theta_A; \mathbf{X}_A)\\ q_A^f(\mathbf{X}_A, t) &= \hat{q}_A^f(V_{AS}, N_{AS}, \theta_A, \theta_S, \operatorname{Grad}\theta_A; \mathbf{X}_A).\end{aligned} \quad (10)$$

Two fundamental requirements have to be fulfilled in constitutive equations for frictional heat [26]:

(a) The frictional heat fluxes in product with the unit normals are nonpositive, i.e. the frictional heat is supplied to the bodies,

$$\mathbf{q}_A^f \cdot \mathbf{n}^+ \leqslant 0 \quad \mathbf{q}_B^f \cdot \mathbf{n}^- \leqslant 0. \quad (11)$$

This restriction follows from an analysis of the second law of thermodynamics, formulated for the system $A \cup B \cup S$. Friction, wear and frictional heat taken as independent processes are the most important assumptions of the analysis.

(b) The following constraint of energy dissipated in friction process must be satisfied:

$$\mathbf{t}_{AS} \cdot \mathbf{V}_{AS} + \mathbf{t}_{BS} \cdot \mathbf{V}_{BS} = \mathbf{c} \cdot \omega + \mathbf{q}_A^f \cdot \mathbf{n}^+ + \mathbf{q}_B^f \cdot \mathbf{n}^- + q_f^* + \beta_A + \beta_B + \beta^*. \quad (12)$$

The amount of the generated frictional heat at the contact is strictly connected with the work done by friction.

In view of equation (12), the frictional power ($\mathbf{t}_{AS} \cdot \mathbf{V}_{AS}, \mathbf{t}_{BS} \cdot \mathbf{V}_{BS}$) is partitioned between the power of microrotations ($\mathbf{c} \cdot \omega$), the frictional heat ($\mathbf{q}_A^f, \mathbf{q}_B^f$) entering into the bodies, the frictional heat supplied to the interfacial layer (q_f^*), energy spent on wear process at the bodies (β_A, β_B) and the energy of wear process at the interfacial layer, e.g. crush process (β^*). In most cases, other forms of energy dissipation are negligibly small. The constraint (12) produces qualitative and quantitative restrictions on the frictional heat description.

The equation (12) does not decide which part of the rub energy is converted into microrotation power of interfacial layer particles, heat and wear energy.

Experiments show that 85–95% of the friction force power transforms into heat [27].

3. CONSTITUTIVE EQUATIONS FOR ANISOTROPIC FRICTIONAL HEAT

In the most simple case the constitutive relation of the frictional heat, equation (10), can be defined as a function of the normal pressure and the sliding velocity,

$$q_A^f(\mathbf{X}_A, t) = -w_A N_{AS} V_{AS} \quad (13)$$

$$w_A = w_A(\theta_A, \theta_B, \mathrm{Grad}\,\theta_A) \quad (14)$$

where, w_A is a frictional heat intensity coefficient, being a function of the temperatures and the gradient of temperature. Both independent variables, i.e. pressure and velocity, do not define directional properties of frictional heat. Therefore, the frictional heat intensity coefficient must describe anisotropic properties of frictional heat.

Let us assume that the frictional heat intensity is a function of a sliding direction parameter α_v, i.e.

$$w_A = w_A(\alpha_v) \quad \alpha_v \in \langle 0, 2\pi \rangle. \quad (15)$$

α_v is a measure of an oriented angle between the unit vector $\mathbf{v}_0$ of a reference direction at the contact, and the sliding velocity unit vector $\mathbf{v}_{AS}$. Furthermore, we postulate that the frictional heat intensity function, equation (15), and an anisotropic friction force coefficient function $\mu_\alpha(\alpha_v)$ are of the same type,

$$w_A(\alpha_v) \sim \mu_\alpha(\alpha_v). \quad (16)$$

The direction of the largest values of friction is simultaneously the direction of the largest values of frictional heating. This follows from the largest values of friction power partitioned between heat and wear for that direction. For another sliding direction, the friction force can be the lowest and the friction force power transformed into heat and wear can be the lowest as well. The postulate (16) ensures an homogeneous change of terms in the constraint of energy dissipated in the friction process, equation (12) [28].

A deviation in the friction force from the direction of sliding, and a dependence of the friction magnitude on the sliding direction, are main features of contacts with anisotropic friction [29, 30]. The friction coefficient μ_α and the coefficient $\mu_\alpha^\perp$ of the friction force component normal to the sliding direction can characterize anisotropic friction. Additionally, the angle β of the friction force inclination to the sliding direction, and a curve drawn by the friction force vectors (hodograph of the friction force), completely define frictional anisotropy [29, 30].

The friction force coefficient μ_α is defined by

$$\mu_\alpha = N_{AS}^{-1}\,|\mathbf{t}_{AS}|\,. \quad (17)$$

The constitutive equation of the anisotropic friction force has the following form:

$$\begin{aligned}\mathbf{t}_{AS} = -N_{AS}\{&[\mathbf{C}_{10} + \mathbf{C}_{11}\cos(n_1\alpha_v) \\ &+ \mathbf{C}_{12}\sin(m_1\alpha_v)]\mathbf{v}_{AS}\ldots \\ &+ [\mathbf{C}_{n0} + \mathbf{C}_{n1}\cos(n_n\alpha_v) \\ &+ \mathbf{C}_{n2}\sin(m_n\alpha_v)]\underbrace{(\mathbf{v}_{AS}\otimes\ldots\otimes\mathbf{v}_{AS})}_{2n-1\ \text{copies}}\end{aligned}$$

$$n_1, m_1, \ldots, n_n, m_n = 0, 1, 2, \ldots. \quad (18)$$

$\mathbf{C}_{ik}$ $(i = 1, \ldots, n; k = 0, 1, 2)$ are constant friction tensors [29, 30]. Components of the slip velocity unit vector are as follows:

$$[\mathbf{v}_{AS}] = [\cos\alpha_v \sin\alpha_v]^{\mathrm{T}}. \quad (19)$$

The friction force equation (18), restricted to the second friction tensors $\mathbf{C}_{1k}$ $(k = 0, 1, 2)$, has the following form in the representative notation:

$$t^i = -N_{AS}[C_0^{ij} + C_1^{ij}\cos(n_1\alpha_v) + C_2^{ij}\sin(m_1\alpha_v)]v_j$$
$$\alpha_v \in \langle 0, 2\pi \rangle \quad i, j = 1, 2. \quad (20)$$

Depending on the form of the friction tensors, we get descriptions of anisotropic friction and anisotropic frictional heat with different numbers of constants and parameters. Using three spherical friction tensors,

$$C_k^{ij} = \mu_k \delta^{ij} \quad k = 0, 1, 2 \quad i, j = 1, 2 \quad (21)$$

the friction force coefficient is a trigonometrical polynomial,

$$\mu_\alpha = \mu_0 + \mu_1\cos(n\alpha_v) + \mu_2\sin(m\alpha_v). \quad (22)$$

According to assumption (16), we obtain the following frictional heat intensity coefficient:

$$w_A(\alpha_v) = w_0 + w_1\cos(n\alpha_v) + w_2\sin(m\alpha_v). \quad (23)$$

$n, m = 0, 1, 2, \ldots$—two parameters; w_0, w_1, w_2—three constants. Taking the first friction tensor in equation (20) with non-zero components on the diagonal, and the second and the third tensors equal to zero, i.e.

$$C_0^{11} = \mu_{(0)1} \quad C_0^{22} = \mu_{(0)2} \quad C_1^{ij} = C_2^{ij} = 0 \quad i, j = 1, 2 \quad (24)$$

we get the following frictional heat intensity coefficient:

$$w_A(\alpha_v) = [(w_1\cos\alpha_v)^2 + (w_2\sin\alpha_v)^2]^{1/2}. \quad (25)$$

w_1, w_2—two constants. Having the first friction tensor spherical and the second and the third tensors equal to zero, i.e.

$$C_0^{ij} = \mu_0\delta^{ij} \quad C_1^{ij} = C_2^{ij} = 0 \quad i, j = 1, 2 \quad (26)$$

the frictional heat coefficient is constant,

$$w(\alpha_v) = w_0. \quad (27)$$

w_0 is the frictional heat constant.

Substituting the constitutive equation (13) into the thermodynamic requirement (11), and taking into account that N_{AS} and V_{AS} are positive, we get the following restriction imposed on the frictional heat intensity:

$$w_A(\alpha_v) \geqslant 0 \quad \forall\alpha_v \in \langle 0, 2\pi \rangle. \quad (28)$$

It restricts values of constants w_0, w_1 and w_2 in the relation (23). The constitutive relation (25) satisfies the inequality (28) for arbitrary values of constants $w_1, w_2 \in R$.

4. PROPERTIES OF THE FRICTIONAL HEAT CONSTITUTIVE EQUATIONS

Property 1

The frictional heat constitutive equations (13) and (15) satisfy the axiom of objectivity.

Two different observers of the sliding at the contact recognize the same frictional heat flux. The constitutive equations (13) and (15) have an invariant form with respect to arbitrary transformation from the full orthogonal group, if the following condition is satisfied:

$$w_A(\tilde{\alpha}_v) = w_A(\alpha_v) \quad \forall \mathbf{R} \in o \tag{29}$$

$$\mathbf{R}^{-1} = \mathbf{R}^{\mathrm{T}} \quad \det \mathbf{R} = \pm 1 \tag{30}$$

for all $\alpha_v \in \langle 0, 2\pi \rangle$. $\tilde{\alpha}_v$ is a measure of an oriented angle between vectors $\mathbf{R}\mathbf{v}_0$ and $\mathbf{R}\mathbf{v}_{AS}$. The orthogonal transformation preserves angles, and the constitutive equation obeys the condition of material objectivity.

Property 2

Different types of anisotropic frictional heat can be distinguished depending on a number of neutral and extremal value directions of frictional heat.

For the neutral direction, the frictional heat is independent of the sense of the sliding direction, i.e.

$$w_A(\alpha_v) = w_A(\alpha_v + \pi). \tag{31}$$

This sliding direction α_v we call the extremal value direction of frictional heat, if it gives extremal values of the frictional heating, i.e. minimum or maximum,

$$w_A(\alpha_v) = \min \{ w_A(\tilde{\alpha}_v) : \tilde{\alpha}_v \in \langle 0, 2\pi \rangle \}$$

or

$$w_A(\alpha_v) = \max \{ w_A(\tilde{\alpha}_v) : \tilde{\alpha}_v \in \langle 0, 2\pi \rangle \}. \tag{32}$$

In the case of isotropic frictional heat, all sliding directions are neutral and extremal value directions.

Property 3

With the aid of symmetry groups, the following types of anisotropic frictional heat can be classified: isotropic, anisotropic, orthotropic, trigonal anisotropic, tetragonal anisotropic, centrosymmetric anisotropic and non-centrosymmetric anisotropic.

Symmetry properties of anisotropic frictional heat can be defined by elements of the symmetry group $G_w \subset o$, i.e. by a set of transformations which map anisotropic frictional heat in the reference state onto an equivalent state. The following relation holds:

$$w_A(\tilde{\alpha}_v) = w(\alpha_v) \tag{33}$$

for all $\alpha_v \in \langle 0, 2\pi \rangle$ and for all generators of the group G_w. $\tilde{\alpha}_v$ denotes the sliding direction parameter after transformation, with the aid of the symmetry elements. Identity (**1**), inversion (**−1**), rotations ($\mathbf{R}_n^{\gamma}, \gamma = 2\pi/n, n = 1, 2, \ldots, \infty$) and mirror reflections are generators of the symmetry groups. Definitions of the symmetry group elements are given in refs. [28–30].

It is easy to imagine a contact with nonequivalent motions in positive/negative directions of sliding. In other words, there are differences in forward and backward sliding. This noncentrosymmetry deals with differences in surface or material behaviour of contacts when sliding in two opposite directions.

Property 4

If the anisotropic frictional heat has a finite number of neutral directions, then mirror reflections are with respect to neutral directions ($\mathbf{J}_{\mathbf{u}_i}$). If there is a finite number of extremal value directions and all sliding directions are neutral, then mirror reflections are with respect to the extremal value directions ($\mathbf{J}_{\mathbf{s}_i}$).

Other properties of frictional heat are similar to those for anisotropic wear, see ref. [28].

In Table 1 types of anisotropic frictional heat are listed by name with transformations defining their symmetry properties. The full orthogonal group is the group of symmetry of isotropic frictional heat. Centrosymmetric anisotropic frictional heat has only a trivial two-element group of symmetry. The symmetry group of orthotropic frictional heat has the trivial subgroup and the subgroup of mirror reflections with respect to planes orthogonal to extremal value directions of frictional heat. Non-centrosymmetric anisotropic frictional heat does not have the inversion **−1**. Trigonal anisotropy is defined by the identity of a three-fold rotation axis and three mirror reflections with respect to neutral directions. Tetragonal anisotropic frictional heat has the identity of a four-fold rotation axis and four mirror reflections with respect to extremal value directions. The presence of symmetries reduces the number of unknown coefficients required to describe the property, and the number of necessary experimental measurements can be reduced. Restrictions imposed on the constants in Table 1 follow from the thermodynamic requirements, equations (11) and (28).

5. COMPOSITIONS OF TWO DIFFERENT FRICTIONAL HEAT ANISOTROPIES

Thus far, we have investigated the contact of two surfaces with anisotropic and isotropic frictional heat properties, or the contact of a surface with anisotropic frictional heat features and a surface of a non-conducting heat body. Next, we study frictional heat at the contact of surfaces with known, different, anisotropic frictional heat properties.

Frictional heat intensity functions of single surfaces can be determined experimentally by sliding a test, third body with isotropic frictional properties. We assume reference directions ($\mathbf{v}_0^A, \mathbf{v}_0^B$) on both con-

Table 1. Types of anisotropic frictional heat

Type of frictional heat	Elements of the symmetry group	Constitutive equation	Independent constants and parameters	Restrictions on the constants
Isotropic	o	w_0	w_0	$w_0 \in R^+$
Orthotropic	$\pm\mathbf{1}, \mathbf{J}_{s_1}, \mathbf{J}_{s_2}$	$[(w_1 \cos\alpha_v)^2 + (w_2 \sin\alpha_v)^2]^{1/2}$ $w_0 + w_1 \cos(2\alpha_v)$	w_1, w_2 w_0, w_1 $n = 2$	$w_1, w_2 \in R$ $w_0 \in R^+; w_1 \in R$ $w_0 \geqslant \|w_1\|$
Centrosymmetric anisotropic	$\pm\mathbf{1}$	$w_0 + w_1 \cos(2\alpha_v) + w_2 \sin(2\alpha_v)$	w_0, w_1, w_2 $n = m = 2$	$w_0 \in R^+; w_1, w_2 \in R$ $w_0 \geqslant -w_1 \cos\alpha_v - w_2 \sin\alpha_v$
Non-centrosymmetric anisotropic	$+\mathbf{1}$ $+\mathbf{1}, \mathbf{J}_{u_1}$	$w_0 + w_1 \cos\alpha_v + w_2 \sin\alpha_v$ $w_0 + w_1 \cos\alpha_v$	w_0, w_1, w_2 $n = m = 1$ w_0, w_1 $n = 1$	$w_0 \in R^+; w_1 \in R$ $w_0 \geqslant \|w_1\|$
Trigonal anisotropic	$+\mathbf{1}, \mathbf{R}_n^{2/3\pi}$, $\mathbf{J}_{u_1}, \mathbf{J}_{u_2}, \mathbf{J}_{u_3}$	$w_0 + w_1 \cos(3\alpha_v)$	w_0, w_1 $n = 3$	
Tetragonal anisotropic	$+\mathbf{1}, \mathbf{R}_n^{\pi/2}, \mathbf{J}_{s_1}$, $\mathbf{J}_{s_2}, \mathbf{J}_{s_3}, \mathbf{J}_{s_4}$	$w_0 + w_1 \cos(4\alpha_v)$	w_0, w_1 $n = 4$	

tacting surfaces. Then the following relation holds between the sliding direction parameters on surfaces A and B:

$$\alpha_v^B = \alpha_v^A - \varphi \tag{34}$$

where φ is an angle of relative position of the contacting surfaces.

It is postulated that for a given normal pressure (N_{AB}) and a relative velocity (V_{AB}) at the contact of two surfaces, a resultant frictional heat flux is equal to the product of a "composition coefficient" (κ) and the sum of frictional heat fluxes obtained for each surface taken separately, i.e.

$$q_{AB}^{f} = \kappa(q_A^{f} + q_B^{f}). \tag{35}$$

Heat flux components q_A^f and q_B^f correspond to frictional heat when sliding the test body along the contacting surfaces. The composition coefficient is an experimental quantity and its value does not affect the description of frictional heat anisotropy.

The resultant frictional heat flux can be defined by

$$q_{AB}^{f} = -w_{AB}(\alpha_v^A) N_{AB} V_{AB}. \tag{36}$$

Substituting equation (36) into equation (35), we obtain the following resultant frictional heat intensity defined with respect to the surface A:

$$w_{AB}(\alpha_v^A) = \kappa[w_A(\alpha_v^A) + w_B(\alpha_v^A - \varphi)] \tag{37}$$

where w_A and w_B are frictional heat intensities of surfaces A and B, respectively.

The symmetry group of the resultant frictional heat at the contact of two surfaces with different frictional heat properties is equal to an intersection (a common part) of symmetry groups for the surfaces A and B, i.e.

$$G_w^{AB} = G_w^A \cap G_w^B \tag{38}$$

where G_w^A and G_w^B describe symmetry features of the frictional heat for surfaces A and B, respectively.

Examples of composition of frictional heat anisotropies can be easily given, using formula (37). Let the frictional heat properties of two surfaces be described in terms of an anisotropic heat coefficient $w_A(\alpha_v^A)$ and an isotropic heat coefficient $w_A = \text{const.}$ Heat properties at this contact are defined in terms of the anisotropic frictional heat intensity

$$w_{AB}(\alpha_v^A) = \kappa[w_A(\alpha_v^A) + w_B]. \tag{39}$$

It does not depend on the relative position of the surfaces. The intersection of the symmetry groups is given by

$$\{\pm\mathbf{1}\} \cap \{o\} = \{\pm\mathbf{1}\}. \tag{40}$$

When both surfaces have isotropic heat properties ($w_A, w_B = \text{const.}$), then heat features of the contact are described by the isotropic heat intensity coefficient,

$$w_{AB}(\alpha_v^A) = \kappa(w_A + w_B) = \text{const.} \tag{41}$$

The common part of two full orthogonal groups is the full orthogonal group. A contact between a surface with isotropic frictional heat properties ($w_A = \text{const.}$) and a surface with orthotropic heat features denoted by the heat coefficient $w_B(\alpha_v^B)$ has orthotropic properties, depending on the angle φ of the relative positions of the contacting surfaces,

$$w_{AB}(\alpha_v^A) = \kappa[w_A + w_B(\alpha_v^A - \varphi)]. \tag{42}$$

This frictional heat orthotropy is defined by the following intersection of the symmetry groups:

Table 2. Compositions of two different frictional heat anisotropies

Surface A	Surface B	Composition result
Isotropic	Isotropic	Isotropic
	Orthotropic	Orthotropic depends on φ
	Centrosymmetric anisotropic	Centrosymmetric anisotropic depends on φ
	Non-centrosymmetric anisotropic	Non-centrosymmetric anisotropic depends on φ
	Trigonal anisotropic	Trigonal anisotropic depends on φ
	Tetragonal anisotropic	Tetragonal anisotropic depends on φ
Centrosymmetric anisotropic	Isotropic	Centrosymmetric anisotropic
	Orthotropic	Centrosymmetric anisotropic depends on φ
	Centrosymmetric anisotropic	Centrosymmetric anisotropic depends on φ
	Non-centrosymmetric anisotropic	Non-centrosymmetric anisotropic depends on φ
	Trigonal anisotropic	Non-centrosymmetric anisotropic depends on φ
	Tetragonal anisotropic	Centrosymmetric anisotropic depends on φ
Trigonal anisotropic	Isotropic	Trigonal anisotropic
	Orthotropic	Non-centrosymmetric anisotropic depends on φ
	Centrosymmetric anisotropic	Non-centrosymmetric anisotropic depends on φ
	Non-centrosymmetric anisotropic	Non-centrosymmetric anisotropic depends on φ
	Trigonal anisotropic	Trigonal anisotropic depends on φ
	Tetragonal anisotropic	Non-centrosymmetric anisotropic depends on φ

$$\{o\} \cap \{\pm \mathbf{1}, \mathbf{J}_{s_1}, \mathbf{J}_{s_2}\} = \{\pm \mathbf{1}, \mathbf{J}_{s_1}, \mathbf{J}_{s_2}\}. \quad (43)$$

In Table 2 are given selected examples of the composition of frictional heat anisotropies of the surfaces A and B. If the composition result depends on the angle φ of the relative positions of the surfaces, then this fact is marked in Table 2 with the aid of the comment "depends on φ". The composition result does not depend on the angle φ if the frictional heat symmetry group of the surface B contains rotations about the normal to the contact ($\mathbf{R}_n^\gamma, \gamma \in \langle 0, 2\pi \rangle$). This is the case of isotropic frictional heat of surface B.

6. ILLUSTRATIVE EXAMPLES OF ANISOTROPIC FRICTIONAL HEAT

It is important to consider the directional dependence of frictional heat, together with anisotropic friction. We study friction and frictional heat properties of a material point (a pin) sliding in a plane (a disc) with isotropic and orthotropic friction and frictional heat.

The motion has an initial velocity oriented to the reference system by an angle α_0. The equation of the sliding motion of the pin,

$$m_0 \ddot{\mathbf{r}} = \mathbf{t}_{AB} \quad (44)$$

$$t = 0 \begin{cases} \mathbf{r}_0 = \mathbf{O} \\ \dot{\mathbf{r}}_0 \neq \mathbf{O} \end{cases} \quad (45)$$

$$[\dot{\mathbf{r}}_0] = [\dot{r}_0 \cos \alpha_0 \quad \dot{r}_0 \sin \alpha_0]^{\mathrm{T}} \quad (46)$$

is solved by means of the Runge–Kutta fourth order method. Figure 3 presents trajectories of the retarded motion of the material point in a plane, with the orthotropic friction illustrated by Fig. 2. Orthotropic friction tensor components are as follows:

$$C_0^{11} = 0.13 \quad C_0^{12} = C_0^{21} = -0.03 \quad C_0^{22} = 0.08.$$

The length of the trajectory depends on the frictional resistance. Intervals between points on the trajectories shown in Fig. 3 correspond to constant time intervals (0.2 s). For isotropic friction, rectilinear trajectories coincide with the initial sliding velocities. From the motion equation solutions we obtain: sliding direction parameter (α_v), slip velocity magnitude ($V_{AB} = |\dot{\mathbf{r}}|$) and time of sliding ($t$).

Frictional heat is generated at the contact between the pin and the plane. The heat conduction equation for a one-dimensional continuum, including heat transfer into the surrounding area, has the following form:

$$\frac{\partial T}{\partial t} = KT_{,zz} - b(T - T_0)$$

$$T = T(z,t) \quad z \in \langle 0, \infty). \quad (47)$$

Initial and boundary conditions of the heat conduction problem are as follows:

$$T(z, t = 0) = T_0$$

$$k \frac{\partial T(z = 0, t)}{\partial z} = q_A^{\mathrm{f}}$$

$$\lim_{z \to \infty} T(z,t) = T_0 \quad (48)$$

where

$$K = \frac{k}{c\rho} \quad (49)$$

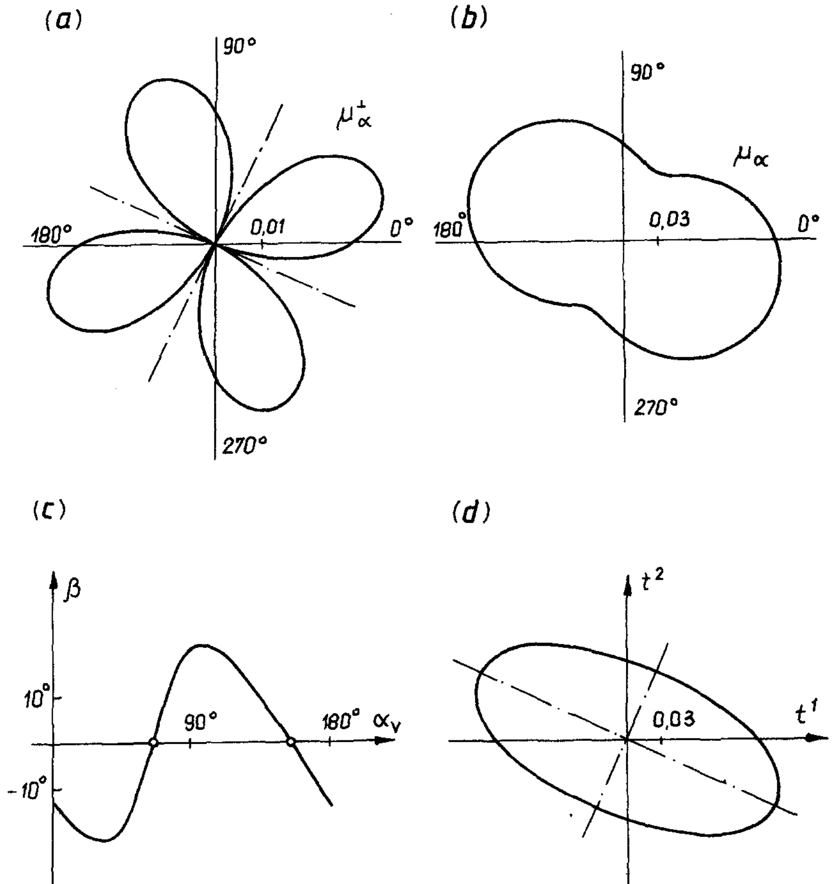


Fig. 2. Illustration of the orthotropic friction: (a) friction coefficient $\mu_\alpha^\perp$; (b) friction coefficient μ_α; (c) deviation angle of the friction force from the sliding direction; (d) friction force hodograph.

$$b = \frac{hg}{c\rho F}. \tag{50}$$

The steel pin is characterized by the following data: mass density $\rho = 7.7 \times 10^3$ kg m^{-3}, specific heat $c = 0.46 \times 10^3$ N m kg^{-1} K^{-1}, thermal conductivity $k = 45.4$ N s^{-1} K^{-1}, thermal diffusivity coefficient $K = 12.5 \times 10^{-6}$ m^2 s^{-1}, pin cross-section area $F = 1 \times 10^{-4}$ m^2, pin circumference $g = 4 \times 10^{-2}$ m. With the aid of the Nusselt number formula, the heat transfer coefficient can be determined. Its value depends on the sliding velocity, and an average value is equal to $h = 32.9$ N m^{-1} s^{-1} K^{-1}.

The heat conduction problem, equations (47) and (48), can be reduced to a simple form by substitution of the following:

$$T - T_0 = \upsilon e^{-bt} \tag{51}$$

where $\upsilon = \upsilon(z, t)$. A solution of the reduced problem is well known, see ref. [31]. Then the solution of the heat conduction equations (47), (48) is given by

$$T(z, t) = T_0 + \frac{q_A^f}{k}\sqrt{\frac{K}{\pi}}\int_0^t \frac{1}{\sqrt{\tau}} e^{-(b\tau + (z^2/4K\tau))}\, d\tau. \tag{52}$$

In the examples, it is assumed that all friction power goes into heat, and all generated heat flows into the pin (the sliding plane is an isolator). Then the constraint of energy dissipated (12) reduces to the following relation:

$$-(\cos\beta) t_{AB} V_{AB} = q_A^f. \tag{53}$$

By substitution of the frictional heat flux and the friction force defined by equations (13) and (17) into equation (53), we obtain the quantitative restriction on the frictional heat intensity coefficient,

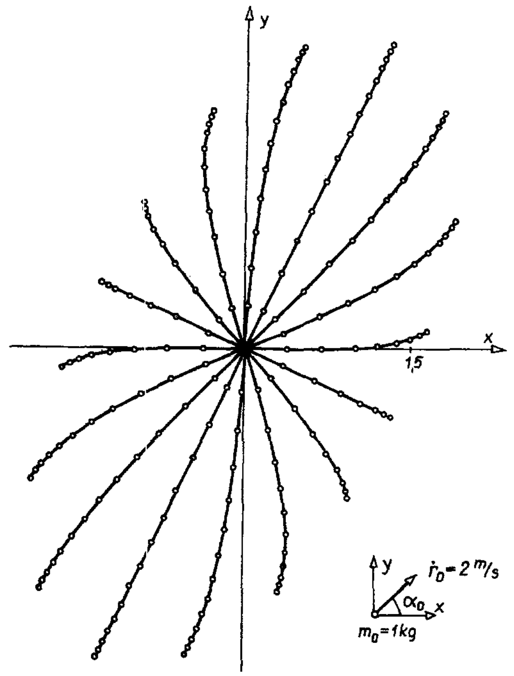
$$w_A(\alpha_v) = \mu_\alpha(\alpha_v) \cos\beta. \tag{54}$$

In the examples, the frictional heat intensity function is the trigonometrical polynomial, equation (23). The anisotropic frictional heat flux vector q_A^f is a function of the time of sliding. We assume that q_A^f is constant for every time step Δt, i.e. for $t \in \langle t_i, t_{i+1} \rangle$, where

$$t_{i+1} = t_i + \Delta t. \tag{55}$$

We apply the same time step Δt in numerical integration of the motion equation (44) and in numerical calculations of the integral (47).

Calculated contact temperatures (i.e. $T(z = 0, t)$



α_0	20°	42,5°	65°	87,5°	110°	132,5°	155°	177,5°
	200°	222,5°	245°	267,5°	290°	312,5°	335°	357,5°

Fig. 3. Motion of a material point in a plane with orthotropic friction.

depend on the type of anisotropic friction and on the type of anisotropic frictional heat. Figures 4–7 illustrate frictional heat intensity coefficient $w(\alpha_v)$ with respect to polar coordinates, and the contact temperatures as sliding time functions (given for selected sliding directions α_0). Graphical representations of the heat intensity functions in Figs. 4–7 are equal to the isotropic friction coefficient function or are similar to the orthotropic friction coefficient function, respectively [the influence of cos β in equation (54) is neglected].

Figures 4 and 5 present the contact temperature for the pin sliding in a plane with isotropic friction properties ($\mu_\alpha = 0.1$; $\dot{r}_0 = 2\,\mathrm{m\,s^{-1}}$). Two different frictional heat intensity functions are taken into account: isotropic ($w_0 = 0.1$), Fig. 4, and orthotropic ($w_0 = 0.105$; $w_1 = 0.039$; $n = 2$), Fig. 5. Motion trajectories in the isotropic friction plane are segments of line of the same length for all sliding directions. Therefore, the contact temperature does not depend on the sliding direction for the isotropic frictional heat intensity function, Fig. 4. It depends on the sliding direction α_0, if the frictional heat intensity is orthotropic, Fig. 5.

Results ploted in Figs. 6 and 7 show the contact temperatures for the pin sliding in the plane with orthotropic friction (see Figs. 2 and 3). Figure 6 illustrates the case of isotropic frictional heat intensity function ($w_0 = 0.1$) and Fig. 7 shows orthotropic heat intensity ($w_0 = 0.105$; $w_1 = 0.039$; $n = 2$). The orthotropic friction induces different sliding trajectories and sliding times. Thus, contact temperatures depend on the sliding direction, in spite of the frictional heat intensity being isotropic (Fig. 6). If both phenomena (friction and frictional heat) are orthotropic, it markedly changes the contact temperatures (Fig. 7). For all sliding directions the same kinetic energy is dissipated in friction and converted into the frictional heat.

Since the pin motion is the retarded motion, then the amount of generated heat decreases if the sliding time increases. The contact temperature plots correspond to the real time of sliding (from the beginning to the end of sliding). We do not consider cooling effects.

Figure 8 presents temperature profiles in the pin (z-distance from the contact) at the time of sliding $t = 1.4$ s, for different sliding directions α_0 and under conditions illustrated by Fig. 7. Heat is conducted into surroundings; the highest temperature occurs at the contact surface.

7. CONCLUSIONS

(a) With the aid of a thermodynamical approach and a postulate that anisotropic friction and frictional heat have common properties, we proposed a method for deriving constitutive relations for anisotropic frictional heat.

(b) Variables and forms of the constitutive relations for frictional heat are restricted by the axiom of objectivity, the second law of thermodynamics, and the constraint of energy dissipated in the friction process.

(c) Constitutive equations for anisotropic frictional heat enable prediction of the behaviour of a contact, that can be confirmed by experimental observations. Mathematical properties of the constitutive equations define a range of possible applications of the anisotropic frictional heat models in investigations of theoretical and practical problems of technology.

Contact surface temperatures are important in machining processes (boring, grinding, polishing, cutting), metal forming, frictional welding, electrical contacts, etc. High working temperatures characterize automative, railway and aircraft brakes. The high surface temperature occurs in turbine blade tip/shroud interactions due to very high sliding speed [17]. The contact temperature can strongly affect the friction, the rate of wear and surface damage processes.

Developments in composite and fiber reinforced materials provide the motivation for research in anisotropic phenomena of friction, wear, and frictional heat. Composite materials are widely used, e.g. in brakes [10]. The recognition and understanding of anisotropic contact phenomena represent one of the

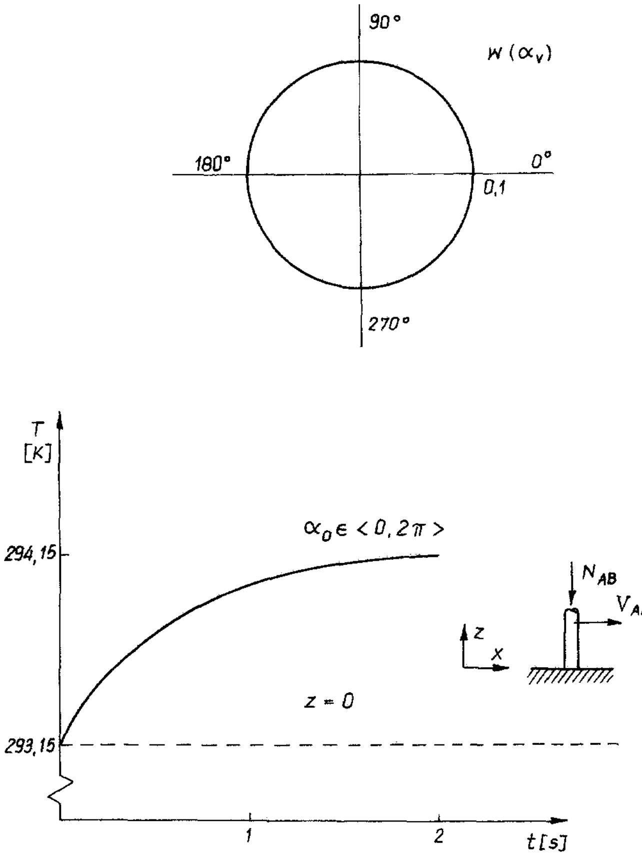


Fig. 4. Isotropic frictional heat intensity function and the contact temperature ($z = 0$) of a steel pin for sliding in a plane with isotropic friction.

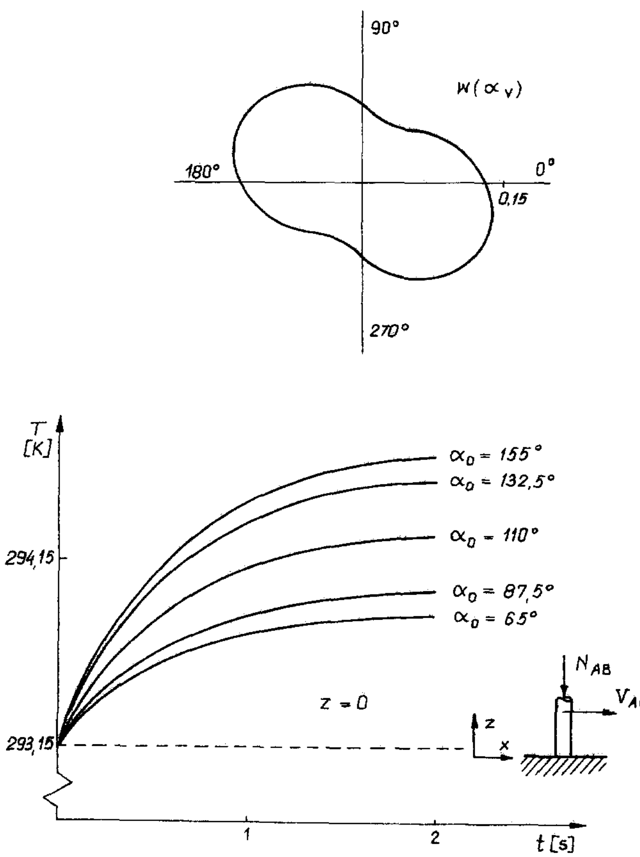


Fig. 5. Orthotropic frictional heat intensity function and the contact temperature ($z = 0$) of a steel pin for sliding in different directions α_0 in a plane with isotropic friction.

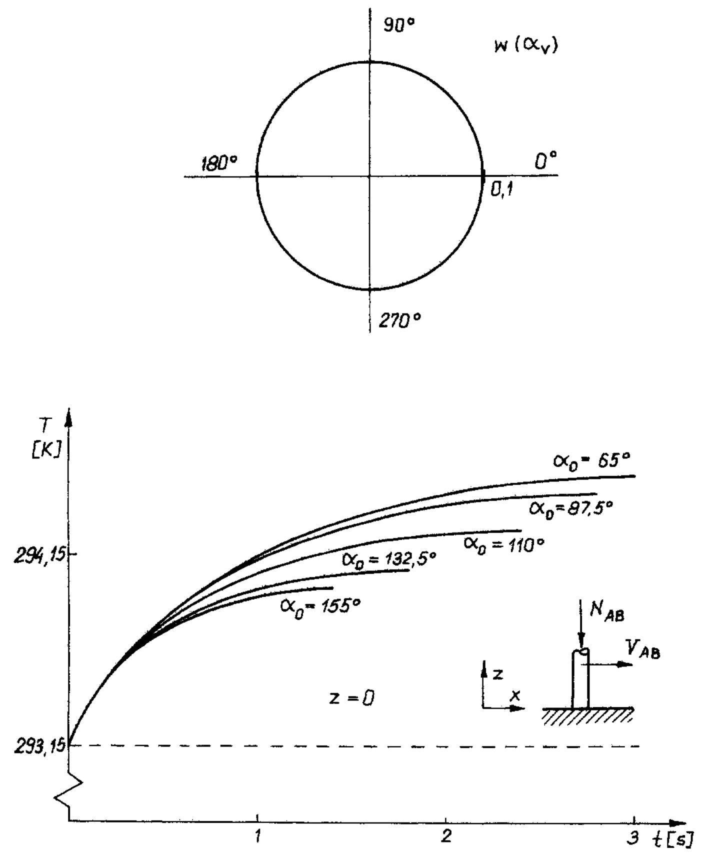


Fig. 6. Isotropic frictional heat intensity function and the contact temperature ($z = 0$) of a steel pin for sliding in different directions α_0 in a plane with orthotropic friction (see Fig. 2).

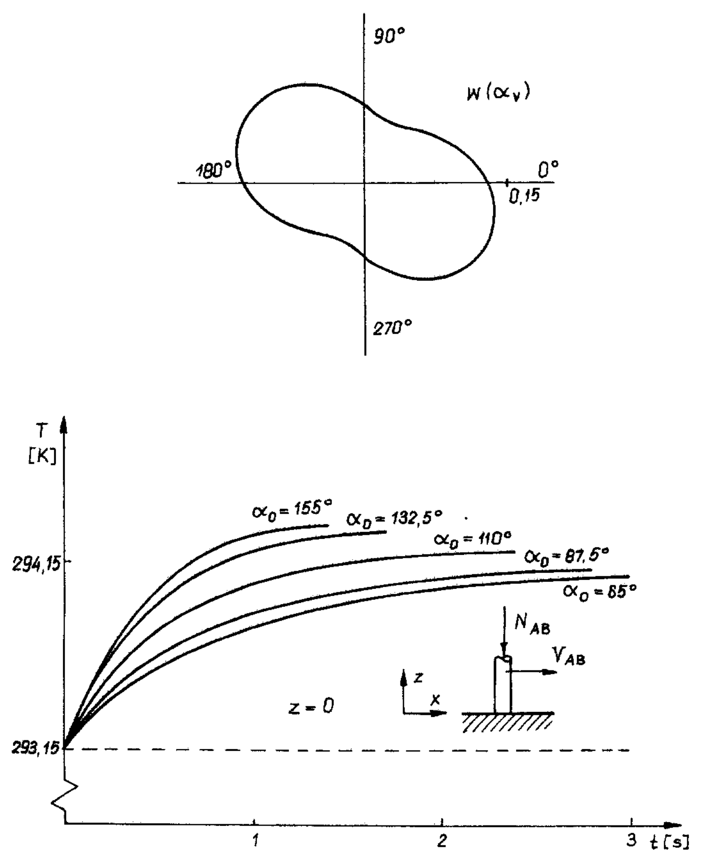


Fig. 7. Orthotropic frictional heat intensity function and the contact temperature ($z = 0$) of a steel pin for sliding in different directions α_0 in a plane with orthotropic friction (see Fig. 2).

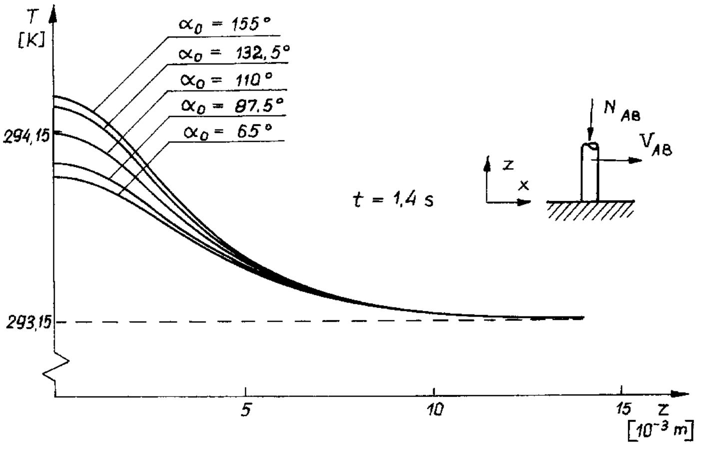


Fig. 8. Temperature profiles in a pin for sliding in different directions α_0 at constant sliding time $t = 1.4$ s and under conditions illustrated by Fig. 7.

major problems of forming and cutting processes. Frictional heat anisotropy can play an important role in engineering applications of monocrystals, e.g. used in technological operations.

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